4.6. Natural Deduction for Categorical Syllogism

1. Rules for Syllogistic Deductions. While various methods suffice to demonstrate that a categorical syllogism is valid, here we develop a test along the lines of our earlier formal deductions – where the conclusion is deduced from the premises through a chain of inferences, each fitting some inference rule.

Such ‘syllogistic deduction’ calls for only two rules: Switching and Linking.

Switching allows the two terms of a categorical sentence to change places: the subject becomes the predicate, the predicate becomes the subject.[1]

To ensure validity of inference, **Switching in univeral sentences** is **restricted**: when terms are switched in a universal sentence, the **value** of each term must be changed. So if a term is negative before being switched, it is positive afterward; if positive before switching, negative after. English examples illustrate.

All **men** are **mortal beings**.

∴ All **non-mortal beings** are **non-men**.

All **lizards** are **non-mammals**.

∴ All **mammals** are **non-lizards**.

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[1] This is very similar to what is traditionally called “conversion,” first treated in Aristotle’s **On Interpretation** Chapter 2.

The general pattern for Universal Switching is like so.

Switching (S): Universal Sentences

All ⬟ are ✱

All non-✱ are non-⬟

All non-⬟ are non-✱

All ✱ are ⬟

All ⬟ are non-✱

All ✱ are non-⬟

All non-⬟ are ✱

All non-✱ are ⬟

In Existential sentences Switching can occur without restriction.

Switching (S): Existential Sentences

Some ⬟ are ✱

Some ✱ are ⬟

English examples illustrate.

Some **men** are **doctors**.

∴ Some **doctors** are **men**.

Some **men** are **non-husbands**.

∴ Some **non-husbands** are **men**.

The second rule, **Linking**, derives a new sentence from two previous sentences. The middle sentence – the **linking premise** – has as its terms the **predicates** of the other sentences. Here are two English examples.

All humans are **mortal beings**.
All **mortal beings** are **creatures requiring food**.
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∴ All humans are **creatures requiring food**.

Some mammals are **non-finned beings**.
All **non-finned beings** are **non-fish**.
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∴ Some mammals are **non-fish**.

Linking (L)

All ⬟ are ❋		Some ⬟ are ❋
All ❋ are ✖	⇐ Linking Premise ⇒	All ❋ are ✖
______________		______________
All ⬟ are ✖		Some ⬟ are ✖

Linking must obey the following two *restrictions*:

- ✓ The **linking premise** must be **universal**.
- ✓ The **conclusion** must have the **same quantity** as **the other** (non-linking) **premise**. (If the other premise is universal, the conclusion must universal; if the other premise is existential, the conclusion must be existential.)

Skeletal examples illustrate the two deduction rules at work.

Example 1: we demonstrate the validity of the following argument form.

~~1. All G are H~~
2. All I are non-H

∴ All G are non-I

The deduction begins with the premises, and "Get" line for the desired conclusion.

1~~. All G are ~~H (Premise)
2. All I are non-H (Premise)
Get: All G are non-I

We apply **Switching** to Line (2).

1~~. All G are ~~H (Premise)
2. All I are non-H (Premise)
Get: All G are non-I
3. All H are non-I (2, S)

Linking leads from Lines (1) and (3) to the desired conclusion – at which point the "Get" line is crossed out.

~~1. All G are H (P~~remise)
2. All I are non-H (Premise)
Get: All G are non-I
3. All H are non-I (2, S)
4. All G are non-I (1, 3, L)

Example 2:

1. All G are non-H
2. All I are H

———————

∴ All G are non-I

1. All G are non-H (Premise)
2. All I are H (Premise)

———————(**Get**: All G are non-I)

3. All non-H are non-I (2, S)
4. All G are non-I (1, 3, L)

For each of the following argument forms, the conclusion can be deduced from the premises using just Switching and/or Linking. (The first four require only Linking.)

1. All G are H . All H are I ∴ All G are I
2. All G are H . All H are non-I ∴ All G are non-I
3. Some G are H . All H are I ∴ Some G are I
4. Some G are H . All H are non-I ∴ Some G are non-I
5. Some G are H . All I are non-H ∴ Some G are non-I
6. Some G are non-H . All I are H ∴ Some G are non-I
7. All H are G . Some H are I ∴ Some G are I
8. Some H are G . All H are I ∴ Some G are I
9. All H are G . Some H are non-I ∴ Some G are non-I
10. Some H are G . All H are non-I ∴ Some G are non-I
11. All H are non-G . All I are H ∴ All G are non-I
12. All H are G . Some I are H ∴ Some G are I
13. Some H are G . All I are non-H ∴ Some G are non-I

2. Arguments Requiring Existence Assumptions. Some arguments in syllogistic form are not valid as stated, but can be made valid by adding an existence premise: an additional premise claiming existence for a certain type of thing. For example, the following argument is not valid.

1. All cyclopses are one-eyed giants
2. All one-eyed giants are living beings

∴ Some cyclopses are one-eyed giants

Recall that "some" is read as: there exists at least one. So while both premises may be true, it is still false that there exists at least one cyclops which is a living being – for in fact there exist no such mythical creatures. The actual world is a validity counterexample for this argument.

However, the conclusion would follow if we add the third premise that "There exist some cyclopses." And that existence assumption can be framed in categorical form as follows.[2]

"There exist some G": Some G are G .

Here we make the essential claim – that there exists something which is G – and then add, repetitively, that this thing is G. The second adds no new information; but the repetition does no harm, and allows the sentence to fit into categorical form.

The following arguments are valid with the added existence assumption (in brackets).

1. All G are H . All H are I . [There are G] ∴ Some G are I
2. All G are H . All H are non-I . [There are G] ∴ Some G are non-I
3. All G are H . All I are non-H . [There are G] ∴ Some G are non-I
4. All H are non-G . All I are H . [There are G] ∴ Some G are non-I
5. All H are G . All H are I . [There are H] ∴ Some G are I
6. All H are G . All I are non-H . [There are H] ∴ Some G are non-I
7. All H are G . All H are non-I . [There are H] ∴ Some G are non-I
8. All H are G . All I are H . [There are I] ∴ Some G are I

[2] This is the ordinary 'I' form, but with the same term serving as subject and predicate. Our statement of sentences in categorical form placed no restriction on subject and predicate that would prevent this; but to those would not recognize this as orthodox categorical form it will mark a slight relaxation of what qualifies as a sentence in categorical form.